2.1 Represent Relations and Functions

**Goal**
- Represent relations and graph linear functions.

### VOCABULARY

**Relation**

**Domain**

**Range**

**Function**

**Equation in two variables**

**Linear function**

### REPRESENTING RELATIONS

A relation can be represented in the following ways:

**Ordered Pairs**
- \((-2, 2)\)
- \((-2, -2)\)
- \((0, 1)\)
- \((3, 1)\)

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>Table</th>
<th>Graph</th>
<th>Mapping Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-2, -2))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((3, 1))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Goal • Represent relations and graph linear functions.

VOCABULARY

Relation A mapping, or pairing, of input values with output values

Domain The set of input values in a relation

Range The set of output values in a relation

Function A relation for which each input has exactly one output

Equation in two variables An equation that has an independent or input variable and a dependent or output variable that depends on the value of the input variable

Linear function A function that can be written in the form \( y = mx + b \), where \( m \) and \( b \) are constants

REPRESENTING RELATIONS

A relation can be represented in the following ways:

<table>
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<tbody>
<tr>
<td>((-2, 2))</td>
<td>(x)</td>
<td>(y)</td>
<td></td>
</tr>
<tr>
<td>((-2, -2))</td>
<td>(-2)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td>(-2)</td>
<td>(-2)</td>
<td></td>
</tr>
<tr>
<td>((3, 1))</td>
<td>(0)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Example 1  Identify functions

Tell whether each relation is a function. Explain.

a. Input  Output
   -2  1
   -1  0
    3  4

b. Input  Output
   -2  2
    1  0
    2  2

Solution

a. The relation \[\text{a function}\] because each input is mapped onto \[\text{output}\].

b. The relation \[\text{a function}\] because the input \[\text{is mapped onto} \] and \[\text{}.\]

Checkpoint  Complete the following exercise.

1. Is the relation given by the ordered pairs \((-5, 2), (-3, -1), (0, 0), (0, 2)\) and \((0, 5)\) a function? Explain.

VERTICAL LINE TEST

A relation is a function if and only if no \[\text{line}\] intersects the graph of the relation at more than \[\text{line}\].
Tell whether each relation is a function. Explain.

<table>
<thead>
<tr>
<th>a. Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b. Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

**Solution**

a. The relation **is** a function because each input is mapped onto **exactly one** output.

b. The relation **is not** a function because the input **2** is mapped onto **2** and **3**.

**Checkpoint** Complete the following exercise.

1. Is the relation given by the ordered pairs \((-5, 2), (-3, -1), (0, 0), (0, 2)\) a function? Explain.
   
   No, the relation is not a function because the value 0 maps to 0, 2, and 5.

**VERTICAL LINE TEST**

A relation is a function if and only if no **vertical** line intersects the graph of the relation at more than **one point**.
Is the relation represented by the graph a function? Explain.

**Solution**

a. This graph ______ represent a function because no vertical line intersects the graph at more than __________.

b. This graph ______ represent a function because the vertical lines at $x = \_\_\_\_$ and at $x = \_\_\_\_$ intersect the graph at more than one point.

**Example 2** Use the vertical line test

To graph an equation in two variables, follow these steps:

Step 1 **Construct** a table of values.

Step 2 **Plot** enough points from the table to recognize a ______.

Step 3 **Connect** the points with a _____ or ______.

**Example 3** Graph an equation in two variables

Graph the equation $y = -2x - 2$.

**Solution**

Step 1 **Construct** a table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>___</td>
<td>___</td>
<td>___</td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>

Step 2 **Plot** the points. Notice that they all lie on a _____.

Step 3 ________ the points with a line.
Example 2  
*Use the vertical line test*

Is the relation represented by the graph a function? Explain.

a.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>2</td>
</tr>
<tr>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>−2</td>
</tr>
<tr>
<td>1</td>
<td>−4</td>
</tr>
<tr>
<td>2</td>
<td>−6</td>
</tr>
</tbody>
</table>

b.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Solution  

a. This graph **does** represent a function because no vertical line intersects the graph at more than **one point**.

b. This graph **does not** represent a function because the vertical lines at $x = 3$ and $x = 6$ intersect the graph at more than one point.

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**GRAPHING EQUATIONS IN TWO VARIABLES**

To graph an equation in two variables, follow these steps:

Step 1 **Construct** a table of **values**.

Step 2 **Plot** enough points from the table to recognize a **pattern**.

Step 3 **Connect** the points with a **line** or **curve**.

---

**Example 3**  
*Graph an equation in two variables*  

Graph the equation $y = −2x − 2$.

Solution  

Step 1 **Construct** a table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>2</td>
</tr>
<tr>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>−2</td>
</tr>
<tr>
<td>1</td>
<td>−4</td>
</tr>
<tr>
<td>2</td>
<td>−6</td>
</tr>
</tbody>
</table>

Step 2 **Plot** the points. Notice that they all lie on a **line**.

Step 3 **Connect** the points with a line.
Tell whether the function is linear. Then evaluate the function when $x = -3$.

a. $f(x) = 6x + 10$

b. $g(x) = 2x^2 + 4x - 1$

**Solution**

a. The function $f$ is __________ because it has the form $f(x) = mx + b$.

$$f(x) = 6x + 10$$

Write function.

$$f(\square) = 6(\square) + 10$$

Substitute $\square$ for $x$.

$$= \square$$

Simplify.

b. The function $g$ is __________ because it has an $x^2$-term.

$$g(x) = 2x^2 + 4x - 1$$

Write function.

$$g(\square) = 2(\square)^2 + 4(\square) - 1$$

Substitute $\square$ for $x$.

$$= \square$$

Simplify.

**Checkpoint** Complete the following exercises.

1. **Exercise**: Tell whether the function is linear. Then evaluate the function when $x = -3$.

   a. $f(x) = 6x + 10$

   b. $g(x) = 2x^2 + 4x - 1$

   **Solution**

   a. The function $f$ is __________ because it has the form $f(x) = mx + b$.

   $$f(x) = 6x + 10$$

   Write function.

   $$f(\square) = 6(\square) + 10$$

   Substitute $\square$ for $x$.

   $$= \square$$

   Simplify.

   b. The function $g$ is __________ because it has an $x^2$-term.

   $$g(x) = 2x^2 + 4x - 1$$

   Write function.

   $$g(\square) = 2(\square)^2 + 4(\square) - 1$$

   Substitute $\square$ for $x$.

   $$= \square$$

   Simplify.

2. **Exercise**: Use the vertical line test to tell whether the relation is a function.

   ![Graph showing a function and a non-function relation]

3. **Exercise**: Graph the equation $y = 2x - 3$.

   ![Graph showing the equation $y = 2x - 3$]

4. **Exercise**: Tell whether the function is linear. Then evaluate the function when $x = -1$.

   a. $f(x) = 2x^3 + 6 - x$

   b. $g(x) = 4x + 9$
Tell whether the function is linear. Then evaluate the function when \( x = -3 \).

\[ a. \quad f(x) = 6x + 10 \quad \quad b. \quad g(x) = 2x^2 + 4x - 1 \]

**Solution**

a. The function \( f \) is **linear** because it has the form \( f(x) = mx + b \).

\[
\begin{align*}
  f(x) &= 6x + 10 \\
  f(-3) &= 6(-3) + 10 \\
  &= -8
\end{align*}
\]

Write function. Substitute \(-3\) for \( x \). Simplify.

b. The function \( g \) is **not linear** because it has an \( x^2 \)-term.

\[
\begin{align*}
  g(x) &= 2x^2 + 4x - 1 \\
  g(-3) &= 2(-3)^2 + 4(-3) - 1 \\
  &= 5
\end{align*}
\]

Write function. Substitute \(-3\) for \( x \). Simplify.

**Checkpoint** Complete the following exercises.

2. Use the vertical line test to tell whether the relation is a function.

\[ \text{is a function} \]

3. Graph the equation \( y = 2x - 3 \).

**Homework**

Tell whether the function is linear. Then evaluate the function when \( x = -1 \).

4. \( f(x) = 2x^3 + 6 - x \)

   not linear; 5

5. \( g(x) = 4x + 9 \)

   linear; 5