Unit 2: Transformations/Rigid Motions

Lesson 9: Congruence in Terms of Rigid Motions

- Students will understand that a congruence between figures gives rise to a correspondence between parts such that corresponding parts are congruent, and they will be able to state the correspondence that arises from a given congruence.
- Students will recognize that correspondences may be set up even in cases where no congruence is present, they will know how to describe and notate all the possible correspondences between two triangles or two quadrilaterals and they will know how to state a correspondence between two polygons.

Opening Exercise

Pictured below are square $ABCD$ and rhombus $GHIJ$. Are they congruent? Explain.

Vocabulary

When figures are congruent, this means that there is a rigid motion (or a composition of rigid motions) that maps the pre-image onto the image. This rigid motion is called a congruence.

Under this definition of congruence, describe why the figures above are not congruent.
Vocabulary

A correspondence between two triangles is a pairing of each vertex of one triangle with one and only one vertex of another triangle. This pairing can be expanded to figures other than triangles and could also involve sides.

Example 1

In the figure below, the triangle on the left has been mapped to the one on the right by a rotation of 240° about $P$. Identify all six pairs of corresponding parts (angles and sides).

![Diagram of triangles]

<table>
<thead>
<tr>
<th>Corresponding angles</th>
<th>Corresponding sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle A \rightarrow$</td>
<td>$AB \rightarrow$</td>
</tr>
<tr>
<td>$\angle B \rightarrow$</td>
<td>$AC \rightarrow$</td>
</tr>
<tr>
<td>$\angle C \rightarrow$</td>
<td>$BC \rightarrow$</td>
</tr>
</tbody>
</table>

a. Is $\triangle ABC \cong \triangle XYZ$? Explain.

b. What rigid motion mapped $\triangle ABC$ onto $\triangle XYZ$? Write the transformation in function notation.

Important Discovery!

Rigid motions produce congruent figures and therefore, congruent parts (angles and sides). As a result, we can say that corresponding parts of congruent figures are congruent.
Exercises

1. \(ABCD\) is a square, and \(AC\) is one diagonal of the square. \(\triangle ABC\) is a reflection of \(\triangle ADC\) across segment \(AC\).

   a. Complete the table below identifying the corresponding angles and sides.

<table>
<thead>
<tr>
<th>Corresponding angles</th>
<th>Corresponding sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\angle BAC \rightarrow)</td>
<td>(AB \rightarrow)</td>
</tr>
<tr>
<td>(\angle ABC \rightarrow)</td>
<td>(BC \rightarrow)</td>
</tr>
<tr>
<td>(\angle BCA \rightarrow)</td>
<td>(AC \rightarrow)</td>
</tr>
</tbody>
</table>

   b. Are the corresponding sides and angles congruent? Justify your response.

   c. Is \(\triangle ABC \cong \triangle ADC\)? Justify your response.

2. Each side of \(\triangle XYZ\) is twice the length of each side of \(\triangle ABC\).

   a. Fill in the blanks below so that each relationship between lengths of sides is true.

   \[
   \underline{\text{________}} \times 2 = \underline{\text{________}}
   \]

   \[
   \underline{\text{________}} \times 2 = \underline{\text{________}}
   \]

   \[
   \underline{\text{________}} \times 2 = \underline{\text{________}}
   \]

   b. Is \(\triangle ABC \cong \triangle XYZ\)? Justify your response

Important Discovery!

Corresponding parts do not always result in congruent figures.
Problem Set

1. Using your understanding of congruence, explain the following:
   
   a. Why is a triangle not congruent to a quadrilateral?

   b. Why is an isosceles triangle not congruent to a scalene triangle?

2. Draw a diagram with two triangles in which all three corresponding angles are congruent but the corresponding sides are not congruent.

3. In the figure below, the triangle on the left has been mapped to the one on the right by a rotation of $80^\circ$ about vertex $C$. Identify all six pairs of corresponding parts (angles and sides).

   ![Diagram](image)

<table>
<thead>
<tr>
<th>Corresponding angles</th>
<th>Corresponding sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle A \rightarrow$</td>
<td>$AB \rightarrow$</td>
</tr>
<tr>
<td>$\angle B \rightarrow$</td>
<td>$AC \rightarrow$</td>
</tr>
<tr>
<td>$\angle C \rightarrow$</td>
<td>$BC \rightarrow$</td>
</tr>
</tbody>
</table>

Write the rigid motion in function notation.