Lesson 5: Translations

- Students learn the precise definition of a translation and perform a translation by construction.

Opening Exercise

You will need a compass and a straightedge

In Unit 1 we learned how to construct parallel lines 2 different ways: using perpendicular lines and by copying an angle.

Let’s review how to construct a parallel line through a point by copying an angle.
Discussion
In order to perform a translation, we will need to use the above construction. Let us investigate the translation. Translations can be done using vectors as shown.

In the figure to the right, quadrilateral $ABCD$ has been translated the length and direction of vector $\overrightarrow{CC'}$. Notice that the distance and direction from each vertex to its corresponding vertex on the image are identical to that of $\overrightarrow{CC'}$.

Notice that each corresponding point can be mapped using the same vector which shows length and direction.

Example 1
Draw the vector that defines each translation below.
Finding the vector is relatively straightforward. Applying a vector to translate a figure is more challenging. To translate a figure, we must construct parallel lines to the vector through the vertices of the original figure and then find the points on those parallel lines that are the same direction and distance away as given by the vector.

Example 2

You will need a compass and a straightedge

In the diagram below, segment $AB$ is translated to produce $A'B'$.

a. Draw the vector that defines this translation.

b. Using your compass, locate $B'$

c. Construct segment $A'B'$
Example 3

You will need a compass and a straightedge

Apply $T_{AB}$ to segment $CD$.

Steps:

1. Draw $C_A$ with radius $AB$.
2. Draw $C_B$ with radius $AC$.
3. Label the intersection point as $C'$.
4. Draw $C_D$ with radius $AB$.
5. Draw $C_B$ with radius $AD$.
6. Label the intersection point as $D'$.
7. Draw segment $C'D'$.
Example 4

You will need a compass and a straightedge

Apply $T_{\overline{AB}}$ to $\triangle XYZ$.

Steps:

1. Draw $C_X$ with radius $\overline{AB}$.
2. Draw $C_Y$ with radius $\overline{AX}$.
3. Label the intersection point as $X'$.
4. Draw $C_Y$ with radius $\overline{AB}$.
5. Draw $C_Y$ with radius $\overline{AY}$.
6. Label the intersection point as $Y'$.
7. Draw $C_Z$ with radius $\overline{AB}$.
8. Draw $C_Z$ with radius $\overline{AZ}$.
9. Label the intersection point as $Z'$.
10. Draw $\triangle X'Y'Z'$.

Lesson Summary

- A translation maps segments onto segments of equal length.
- A translation maps angles onto angles of equal measure.
Problem Set

Translate each figure according to the instructions provided.

1. 2 units down and 3 units left.
   Draw the vector that defines the translation.

2. 1 unit up and 2 units right.
   Draw the vector that defines the translation.

3. Apply $T_{2,-3}$ to translate $\triangle ABC$. 