Lesson 4: Rotations, Reflections, and Symmetry

- Students learn the relationship between a reflection and a rotation.
- Students examine rotational symmetry within an individual figure.

Opening Exercise

\( \Delta A'B'C' \) is the image of \( \Delta ABC \) after a reflection across line \( l_1 \).

a. Reflect the image across line \( l_2 \).

b. What is the relationship between the original triangle and the twice-reflected image?

c. What does point \( R \) represent?

d. How could we determine the angle of rotation?

Reflecting a figure twice over intersecting lines will give the same result as a rotation about the point of intersection!
Example 1

Looking at the Opening Exercise, we can see that the lines of reflection are also **lines of symmetry**. The line of symmetry is equidistant from all corresponding pairs of points.

In the figures below, sketch all the lines of symmetry:

![Diagram of lines of symmetry for a triangle, rectangle, hexagon, and parallelogram.]

Example 2

How can we locate the center of rotation precisely?

Identify the center of rotation in the equilateral triangle $\triangle ABC$ below and label it $D$.

![Diagram of an equilateral triangle with labeled vertices A, B, and C.]
Example 3

**Rotational Symmetry** is a rotation that maps a figure back on to itself.

In **regular polygons** (polygons in which all sides are congruent, the number of rotational symmetries is equal to the number of sides of the figure.

How can we find the angles of rotation?

<table>
<thead>
<tr>
<th></th>
<th>Equilateral Triangle</th>
<th>Square</th>
<th>Regular Pentagon</th>
<th>Regular Hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td># of sides</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angles of Rotation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A rotation of $360^\circ$ will always map a figure back on to itself. This is called the **identity transformation** or **identity symmetry**.

Example 3

When reflecting an object through a point, the image and the pre-image create **point symmetry**. With point symmetry, the object will look exactly the same upside down! This can be seen by reflecting an object through the origin as pictured to the right.

Which of the objects below have point symmetry?

![Image of objects with point symmetry]
Exercises

Using regular pentagon ABCDE pictured to the right, complete the following:

1. Draw all lines of symmetry.

2. Locate the center of rotational symmetry.

3. Describe all symmetries explicitly.
   a. What kinds are there?
   b. How many are rotations?
   c. What are the angles of rotation?
   d. How many are reflections?

4. If the pentagon is rotated clockwise around its center, what is the minimum number of degrees it must be rotated to be an identity transformation?
Problem Set

1. Using the figure to the right, describe all the symmetries explicitly.
   a. How many are rotations?

   b. What are the angles of rotation?

   c. How many are reflections?

   d. Shade the figure so that the resulting figure only has 3 possible rotational symmetries.

Use the figure to answer the questions below.

2. Draw all lines of symmetry. Locate the center of rotational symmetry.

3. Describe all symmetries explicitly.
   a. What kinds are there?

   b. How many rotations? (Include a “360” rotational symmetry,” i.e., the identity symmetry.)